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Effective supergravity descriptions of superstring cosmology

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Abstract

This paper is a review of aspects of supergravity theories that are relevant in superstring cosmology. In particular, it considers the possibilities and restrictions for 'uplifting terms', i.e., methods to produce de Sitter vacua. We concentrate on $N = 1$ and $N = 2$ supergravities, and the tools of superconformal methods, which clarify the structure of these theories. Cosmic strings and embeddings of target manifolds of supergravity theories in others are discussed briefly at the end.

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1. Introduction

Many people studying string theory nowadays study the landscape of vacua that are possible in the context of the different string theories and related to different compactifications of the ten-dimensional spacetime. The resulting vacua each lead to an effective supergravity describing the low-energy physics close to this vacuum. In this sense, the landscape of string theories is a landscape of supergravity theories.

The basic superstring theories already have a supergravity as field theory approximation. After the choice of a compact manifold one is left with an effective lower dimensional supergravity whose number of supersymmetries is determined by the Killing spinors of the compact manifold. Fluxes on branes involved in the string set-up and non-perturbative effects lead to 'gauged supergravities'. These are supergravity theories that are not only the gauge theory of supersymmetry, but also of an extra ordinary Lie group. The combination of supersymmetry with these ordinary gauge symmetries requires some special care.

Though we have discussed supergravity here in the context of superstring theories, the reader should be warned that not every supergravity theory can (so far as we know nowadays) be obtained from a superstring set-up. The set of supergravities that is related to superstring theories has been enlarged in different ways in the past. For example, there was a time when

Table 1. Superalgebras with bosonic subalgebra a direct product of (anti) de Sitter algebra and R-symmetry.

AdS	Superalgebra		R-symmetry
$D=4$	$OSp(N\|4)$		$SO(N)$
$D=5$	$SU(2,2\|N)$	$N\neq 4:$	$SU(N)\times U(1)$
		$N=4:$	$SU(4)$
$D=6$	$F^2(4)$		$SU(2)$
$D=7$	$OSp(8^*\|N)$	N even:	$USp(N)$
dS	**Superalgebra**		**R-symmetry**
$D=4$	$OSp(m^*\|2,2)$	$m=2$	$SO(1,1)$
		$m=4$	$SU(1,1)\times SU(2)$
		$m=6$	$SU(3,1)$
		$m=8$	$SO(6,2)$
$D=5$	$SU^*(4\|2n)$	$n=1$	$SO(1,1)\times SU(2)$
		$n=2$	$SO(5,1)$
$D=6$	$F^1(4)$		$SU(2)$

one did not know how any gauged supergravity would be obtained from superstring theory. It is an open question whether in the future every supergravity theory can be given an embedding in a superstring theory, but it does not look like that will be the case.

In section 2 we will repeat the problems with producing de Sitter vacua in supergravity and the related issue of uplifting terms. Section 3 will give an overview of the various supergravities. But we will then further concentrate on $N = 1$ (and sometimes $N = 2$) and give the general structure of these theories in section 4. A way in which the structure of these supergravities can be understood is the superconformal method, as we explain in section 5. The final remarks in section 6 mention the application of supergravity for the construction of effective theories of cosmic strings and the issue of embedding of a smaller supergravity theory in a larger one.

2. Cosmology and uplifting terms

Supergravity faced already from the early dates a main problem for its application in cosmology. Cosmological constants can easily be produced, but its natural scale would be of the order of the fourth power of the gravitational constant, which is an order of 10^{120} too large, a 'world record of discrepancy between theory and experiment'. Apart from this problem, whose resolution needs massive parameters of another scale, another main problem is the sign of the cosmological constant. Indeed, we have to give up the idea of having a supersymmetric vacuum state. This statement is by now well known, but let us repeat the argument.

It follows from algebraic considerations. If supersymmetry is preserved, there should be a superalgebra of isometries in the vacuum state. This superalgebra should contain the de Sitter algebra if the cosmological constant is positive. Such superalgebras have been classified, and let us compare the situation for anti-de Sitter superalgebras with de Sitter superalgebras in table 1 [1]. The de Sitter superalgebras [2–4] have typically a non-compact R-symmetry subalgebra[1], which leads to non-definite signs in the kinetic terms, and hence to ghosts. Therefore, de Sitter vacua can occur in physical supersymmetric theories only in a

[1] We mention here the superalgebras that are of Nahm's type [5], i.e., where the bosonic subgroup is a direct product of the de Sitter algebra and another simple group, called R-symmetry. More general de Sitter superalgebras have

phase where supersymmetry is completely broken. This might even be welcome in view of the fact that supersymmetry breaking is anyhow necessary to make contact with reality.

As mentioned in the introduction, supergravity is the field theory corresponding to superstring theory. For calculations with string theories it is useful to find an effective supergravity description. If one wants to describe realistic cosmological models, one needs 'uplifting' terms' to raise the value of the cosmological constant. This is, e.g., one of the main issues in the KKLT [8] models. Another example is the effective theory of the cosmic string.

The Abrikosov–Nielsen–Olesen string model has a vector field $W_\mu(x)$ and a complex scalar $\phi(x)$, charged under the gauge symmetry of the vector field (coupling constant g). It uses as potential

$$V = \tfrac{1}{2}D^2 \equiv \tfrac{1}{2}g^2(\xi - \phi^*\phi)^2, \tag{2.1}$$

depending on a constant ξ. The configuration is independent of one of the three spatial directions, and, using polar coordinates (r, θ) in the remaining 2-plane, it is of the form

$$\phi(r,\theta) = |\phi|(r)\,\mathrm{e}^{\mathrm{i}\theta}, \qquad g W_\mu\,\mathrm{d}x^\mu = \alpha(r)\,\mathrm{d}\theta, \tag{2.2}$$

where the function $|\phi|(r)$ is zero at $r = 0$, and thus $D = g\xi$ at that point, while it goes to a constant value $\sqrt{\xi}$ at infinity, leading to a vanishing D. The vector is determined by the requirement that the field strength in the plane directions, F_{12}, is equal to D. When this model is considered in the context of supersymmetry, it is a $1/2$ BPS solution [9, 10]. This model can be embedded in supergravity [11, 12] (see [13, 14] for 3 spacetime dimensions), due to a conspiracy of the spin connection and the R-symmetry connection.

The model can then be seen as the final state after the D3-brane–anti-D3-brane annihilation, which leads to a D1 string. The field ϕ is the tachyon field, and the so-called Fayet–Iliopoulos term ξ represents the brane–antibrane energy [11]. In general, it leads to a positive term in the potential, which is the uplifting that we need in this case in an effective supergravity model.

3. Supergravities

Supergravity exists in many variants. We restrict here to theories where the terms in the action are at most quadratic in spacetime derivatives. Table 2 plots the possible supergravities for dimensions larger or equal to 4. A longer discussion on this table and an extension thereof is given in [15]. The theories with 16 or less real components of the supersymmetry operator allow different additions of matter multiplets to the supergravity multiplet. Suppose now that we have selected an entry in this table. Thus we have chosen a dimension D and a number of supersymmetries (N or Q as you like). Furthermore, assume that for $Q \leqslant 16$, one has also specified the number and type of extra multiplets. For example, in $D = 4$, $N = 1$ one already tells that one wants a theory with 1 vector multiplet and 2 chiral multiplets. How far is the theory then already fixed? In other words, what has still to be determined in order to completely specify the action? The answer is different depending on the range of the number Q.

$32 \geqslant Q > 8$. In this case, the kinetic terms of all the fields are already fixed. The only extra information that one needs is the symmetry group that is gauged by the vector fields and its action on the scalars. Once this is known, the full action is fixed. In particular, the scalar potential of the theory depends on this gauging.

been classified in [6, 7]. But they have not been realized in concrete models, probably because of the appearance of symmetries that are not in the de Sitter algebra and do not commute with it.

Table 2. Supersymmetry and supergravity theories in dimensions 4 to 11. An entry represents the possibility to have supergravity theories in a specific dimension D with the number of (real) supersymmetries indicated in the top row. At the bottom is indicated whether these theories exist only in supergravity, or also with just rigid supersymmetry.

D	32		24	20	16		12	8	4
11	M								
10	IIA	IIB			I				
9	$N=2$				$N=1$				
8	$N=2$				$N=1$				
7	$N=4$				$N=2$				
6	$(2,2)$		$(2,1)$		$(1,1)$	$(2,0)$		$(1,0)$	
5	$N=8$		$N=6$		$N=4$			$N=2$	
4	$N=8$		$N=6$	$N=5$	$N=4$		$N=3$	$N=2$	$N=1$
	SUGRA				SUGRA/SUSY		SUGRA	SUGRA/SUSY	

Table 3. Scalar geometries in theories with more than eight supersymmetries ($4 \leqslant D \leqslant 9$).

D	32	24	20	16		12
9	$\frac{S\ell(2)}{SO(2)\otimes O(1,1)}$			$\frac{O(1,n)}{O(n)}\otimes O(1,1)$		
8	$\frac{S\ell(3)}{SU(2)}\otimes\frac{S\ell(2)}{U(1)}$			$\frac{O(2,n)}{U(1)\times O(n)}\otimes O(1,1)$		
7	$\frac{S\ell(5)}{USp(4)}$			$\frac{O(3,n)}{USp(2)\times O(n)}\otimes O(1,1)$		
6	$\frac{O(5,5)}{USp(4)\times USp(4)}$	$\frac{SO(5,1)}{SO(5)}$		$\frac{O(4,n)}{O(n)\times SO(4)}\otimes O(1,1)$	$\frac{O(5,n)}{O(n)\times USp(4)}$	
5	$\frac{E_6}{USp(8)}$	$\frac{SU^*(6)}{USp(6)}$		$\frac{O(5,n)}{USp(4)\times O(n)}\otimes O(1,1)$		
4	$\frac{E_7}{SU(8)}$	$\frac{SO^*(12)}{U(6)}$	$\frac{SU(1,5)}{U(5)}$	$\frac{SU(1,1)}{U(1)}\times\frac{SO(6,n)}{SU(4)\times SO(n)}$		$\frac{SU(3,n)}{U(3)\times SU(n)}$

$Q = 8$. In this case, apart from the gauging, also the kinetic terms can still vary. These kinetic terms are restricted, but can still depend on some arbitrary prepotential function. The geometry determined by the kinetic terms of the scalars falls in a restricted scheme, which is called 'special geometry'. Once one has chosen the particular special geometry and the gauging, the action (and as such, e.g., the scalar potential) is fixed.

$Q = 4$. Also in this case the gauge group and its action on scalars has to be determined. Moreover, some arbitrary functions determine the kinetic terms. For example, for the chiral multiplets there is the Kähler potential, and for the vector multiplets the kinetic terms are determined by a holomorphic functions of the complex scalars of the chiral multiplets. In this case, the potential depends moreover on a superpotential function W, and in some cases (Abelian gauge groups) on arbitrary constant 'Fayet–Iliopoulos' constants as the ξ in (2.1). There are consistency conditions between the choice of these different ingredients. The gauging and the Fayet–Iliopoulos constants should be compatible in some way with the choice of kinetic terms and superpotential.

To illustrate that the restrictions on the kinetic terms, we present here table 3 of scalar manifolds in theories with more than eight real supercharges. The theories are ordered as in table 2. For more than 16 supersymmetries, there is only a unique scalar manifold, while for 16 and 12 supersymmetries there is a number n indicating the number of vector multiplets that are included.

For 8 supersymmetries, the table is

$D=6$	$D=5$	$D=4$
$\frac{O(1,n)}{O(n)}\times QK$	$VSR\times QK$	$SK\times QK$

(3.1)

Table 4. Multiplets of particles in $N = 1$ and $N = 2$ in 4 dimensions.

$N = 1$			$N = 2$		
Graviton multiplets	$(2, \frac{3}{2})$		Graviton multiplet	$(2, \frac{3}{2}, \frac{3}{2}, 1)$	
Vector multiplets	$(1, \frac{1}{2})$		Vector multiplets	$(1, \frac{1}{2}, \frac{1}{2}, 0, 0)$	Special Kähler
Chiral multiplets	$(\frac{1}{2}, 0, 0)$	Kähler	Hypermultiplets	$(\frac{1}{2}, \frac{1}{2}, 0, 0, 0, 0)$	Quaternionic-Kähler

Here, 'VSR' stands for 'very special real' geometry, 'SK' for special Kähler geometry and 'QK' for quaternionic-Kähler geometry. This thus only specifies a geometric class of manifolds, but they are not uniquely determined.

We now look at the other issue—the gauge group. The number of generators of the gauge group is equal to the number of vector fields[2]. This counting includes as well vectors in the supergravity multiplet and those in vector multiplets. In fact, in general their kinetic terms are mixed and thus they do not have to be distinguished. The gauge group is in principle arbitrary, but to have positive kinetic terms gives restrictions on possible non-compact gauge groups for any supergravity. The symmetries that they gauge act on the other fields. In particular, it is (a subgroup of) the isometry group of the scalar manifold.

For most applications in cosmology the final effective theory has only four or eight real supersymmetries. We will now focus our attention to 4 dimensions, and thus to $N = 1$ or $N = 2$ theories, as well as to the geometry that describes the kinetic terms of the scalars.

4. $N = 1$ and $N = 2$ supergravities

The multiplets of particles with spin up to 2 that can be considered in these theories are given in table 4. The actions of the bosonic sector look like (with ∇ the gauge-covariant derivative)

$$\begin{aligned}
e^{-1}\mathcal{L}_{N=1}|_{\text{bosonic}} &= \tfrac{1}{2}R - \tfrac{1}{4}(\operatorname{Re} f_{\alpha\beta})F^{\alpha}_{\mu\nu}F^{\mu\nu\beta} + \tfrac{1}{8}\,e^{-1}\varepsilon^{\mu\nu\rho\sigma}(\operatorname{Im} f_{\alpha\beta})F^{\alpha}_{\mu\nu}F^{\beta}_{\rho\sigma} \\
&\quad - g_{i\bar{\jmath}}(\nabla_\mu\phi^i)(\nabla^\mu\bar{\phi}^{\bar{\jmath}}) - \mathcal{V}_{N=1}, \\
e^{-1}\mathcal{L}_{N=2}|_{\text{bosonic}} &= \tfrac{1}{2}R + \tfrac{1}{4}(\operatorname{Im}\mathcal{N}_{IJ})F^{I}_{\mu\nu}F^{\mu\nu J} + \tfrac{1}{8}\,e^{-1}\varepsilon^{\mu\nu\rho\sigma}(\operatorname{Re}\mathcal{N}_{IJ})F^{I}_{\mu\nu}F^{J}_{\rho\sigma} \\
&\quad - g_{\alpha\bar{\beta}}(\nabla_\mu z^\alpha)(\nabla^\mu\bar{z}^{\bar{\beta}}) - \tfrac{1}{2}g_{XY}\nabla_\mu q^X\nabla^\mu q^Y - \mathcal{V}_{N=2}.
\end{aligned} \tag{4.1}$$

For $N = 1, \alpha$ labels the vector multiplets, and $i, \bar{\imath}$ label the complex scalars. For $N = 2$, the complex scalars of the vector are labelled by $\alpha, \bar{\alpha} = 1, \ldots, n_V$. The n_V vectors combine with the vector of the gravity multiplet, leading to the label $I = 0, 1, \ldots, N_V$. The $4n_H$ real scalars of the hypermultiplets are labelled by the index X. In (4.1), $g_{i\bar{\jmath}}$, $g_{\alpha\bar{\beta}}$ and g_{XY} are thus the metrics of the scalar manifolds mentioned in table 4. $f_{\alpha\beta}$ are holomorphic functions of the scalars ϕ^i, which should just satisfy some consistency conditions with the gauge group. But the analogous functions $\mathcal{N}_{IJ}$ are complex functions that are determined already by the special Kähler geometry. This brings us to the last terms—the scalar potentials.

For $N = 1$, the potential is determined by a holomorphic superpotential $W(\phi)$ and by the gauging. The latter means that the action of the gauge group on the scalar manifold determines part of the potential. It was mentioned above that for $N \geqslant 2$ the potential is only determined by gauging. This means here that it can be written as a function of the way in which the gauge group acts on hypermultiplets and vector multiplet scalars, encoded in the 'Killing vectors'. Vector multiplet scalars are by the structure of the multiplet in the adjoint of the gauge group. That determines already their Killing vectors. For the hypermultiplets, the manifold can have a

[2] One could say less or equal, but considering that any vector transforms at least as $\delta W_\mu = \partial_\mu\alpha$, even if the α transformation does not act on anything else, we can say that there is a $U(1)$ factor.

group of isometries, of which a subgroup can be gauged by the vectors of the vector multiplets. It is a general fact (Ward identity) in supersymmetry that the scalar potential can be written as a sum of the square of the scalar part of the supersymmetry transformations of the fermions, where the way in which the square must be taken is determined by the metric [16–18]. We illustrate this here for $N = 1$ supergravity. The potential can be written as

$$V = -3M_P^{-2} F_0 \bar{F}_0 + F_i g^{i\bar{\jmath}} \bar{F}_{\bar{\jmath}} + \tfrac{1}{2} D^\alpha (\mathrm{Re}\, f_{\alpha\beta}) D^\beta, \tag{4.2}$$

where M_P is the Planck mass, F_0 appears in the supersymmetry transformation of the gravitino, F_i in that of the chiral fermions and D^α in that of the gaugini as in

$$\begin{aligned} \delta\psi_{\mu L} &= \left(\partial_\mu + \tfrac{1}{4}\omega_\mu{}^{ab}(e)\gamma_{ab} + \tfrac{1}{2}\mathrm{i}A_\mu^B\right)\epsilon_L + \tfrac{1}{2}M_P^{-2}\gamma_\mu F_0 \epsilon_R, \\ \delta\chi_i &= \tfrac{1}{2}\hat{\not\partial}\,\phi_i \epsilon_R - \tfrac{1}{2}F_i\epsilon_L, \qquad \delta\lambda^\alpha = \tfrac{1}{4}\gamma^{\mu\nu}F_{\mu\nu}^\alpha \epsilon + \tfrac{1}{2}\mathrm{i}\gamma_5 D^\alpha \epsilon. \end{aligned} \tag{4.3}$$

The quantities A_μ^B, $\omega_\mu{}^{ab}(e)$, and the $\hat{\partial}$ depend on fields in a way that is not relevant here (see, e.g., [12]). The first two terms in (4.2) are called the F-terms and the last term is the D-term. The former depend on the superpotential W (and on the Kähler potential at order M_P^{-2}). The D-term depends on the gauge transformations of the scalars (and also on the Kähler potential) and can depend on arbitrary Fayet–Iliopoulos constants ξ_α for $U(1)$ factors.

5. Superconformal methods

The superconformal method offers a simplification of supergravity by using a parent rigid supersymmetric theory. A conceptual difference between supersymmetry and supergravity is that the concept of multiplets is clear in supersymmetry but they become mixed in supergravity. This makes superfields an easy tool for rigid supersymmetric theories, but much more complicated for supergravity. The main idea of the superconformal methods is that the supergravity theory can be obtained starting by a rigid supersymmetric theory that has conformal symmetry. This conformal symmetry becomes part of a superconformal group that is gauged. Then, the extra conformal symmetries (and their superpartner symmetries) are gauge-fixed. Before the gauge fixing, everything looks like in rigid supersymmetry with covariantizations. After the gauge fixing it becomes a Poincaré supergravity theory.

This is the generalization of starting with the Lagrangian

$$\mathcal{L} = -\tfrac{1}{2}\sqrt{g}\phi\,\Box^C\phi = -\tfrac{1}{2}\sqrt{g}\phi\,\Box\,\phi + \tfrac{1}{12}\sqrt{g}R\phi^2. \tag{5.1}$$

The conformal covariant D'Alembertian contains the well-known $R\phi^2$ term. This starting action is invariant under local conformal transformations, which scale the metric and also the scalar as $\delta\phi(x) = \Lambda_D(x)\phi(x)$. Hence one can fix the value of this scalar as gauge fixing of these dilatations. A convenient value is $\phi = \sqrt{6}M_P$, which reduces the Lagrangian (5.1) to the Einstein–Hilbert form:

$$\mathcal{L} = \frac{M_P^2}{2}\sqrt{g}R. \tag{5.2}$$

The gauge-fixed value of the scalar thus determines the Planck scale. Remark that to start this example we started with the action of the scalar with a sign such that the kinetic terms are negative definite. This leads to positive kinetic energy of the graviton. In general, when there are more scalars, one has to start with a theory with signature of the kinetic energies $(- + \cdots +)$, and the gauge fixing procedure will remove the negative signature scalar.

To generalize this to supersymmetry, we have to extend the conformal group of transformations to a superconformal group. This enlarges also the set of bosonic symmetries. Let us concentrate here on $N = 1$ in 4 dimensions. In that case, this superconformal group includes apart from the conformal group (including the dilatations as above) also a $U(1)$ R-symmetry. The name R-symmetry refers to the fact that this symmetry does not commute with supersymmetry. To obtain Poincaré supergravity one has then to start with a Weyl multiplet and a compensating chiral multiplet. The Weyl multiplet contains the set of gauge fields of the superconformal algebra. It is the generalization of the $g_{\mu\nu}$ field above. The chiral multiplet is the generalization of the field ϕ in the above example. In this chiral multiplet sits a complex field, which we denote by Y, whose modulus is now fixed by gauge fixing of dilatations and its phase is fixed by the gauge fixing of the mentioned $U(1)$.

When one wants to describe the coupling of n chiral multiplets to supergravity, one starts with rigid supersymmetry where $n+1$ chiral multiplets appear. For rigid supersymmetry, the kinetic terms of the scalars should define a Kähler manifold. Now we impose that there is a conformal symmetry. Technically, this is phrased as the presence of a 'closed homothetic Killing vector' k. Then the structure of the Kähler manifold implies automatically another Killing vector Jk, where J is the complex structure of the manifold. This is gauged by the gauge field of the $U(1)$ in the Weyl multiplet. The gauge fixing of the dilatations and of the $U(1)$ then lead to a n-dimensional Hodge–Kähler manifold, which is the geometric structure of chiral multiplets coupled to supergravity. More details can be found, e.g., in [12, 19, 20].

As an example of the simplifications and the way in which the conformal set-up clarifies the structure of the theory, let us consider the scalar potential. The F-term is

$$\begin{aligned} V_F &= \mathrm{e}^{(K/M_P^2)}\left[-3M_P^{-2}W\bar{W} + (D_iW)g^{i\bar{j}}(D_{\bar{j}}\bar{W})\right],\\ D_iW &\equiv \partial_i W + M_P^{-2}(\partial_i K)W, \end{aligned} \tag{5.3}$$

where K is the Kähler potential and W the superpotential in the supergravity formulation. In the superconformal set-up this is unified by denoting all the scalars as Z^A, which thus includes the compensator Y and the other scalars z^i. Then it is

$$\begin{aligned} V_F &= (\partial_A\mathcal{W})G^{A\bar{B}}(\partial_{\bar{B}}\mathcal{W})|_{\mathcal{K}=-3M_P{}^2}, \qquad & \mathcal{K} &= -3Y\bar{Y}\,\mathrm{e}^{-K(z,\bar{z})/(3M_P{}^2)},\\ G_{A\bar{B}} &= \partial_A\partial_{\bar{B}}\mathcal{K}, & \mathcal{W} &= Y^3M_P^{-3}W(z). \end{aligned} \tag{5.4}$$

Here $\mathcal{K}$ is the Kähler potential and $\mathcal{W}$ is the superpotential of the rigid theory, and the gauge fixing of dilatations has fixed the value of $\mathcal{K}$ to $-3M_P^2$. A similar simplification occurs for the value of the D-term in (4.2) (assuming here a gauge-invariant Kähler potential)

$$\begin{aligned} D^\alpha &= (\mathrm{Re}\, f_{\alpha\beta})^{-1}\mathcal{P}_\beta,\\ \mathcal{P}_\alpha &= \tfrac{1}{2}\mathrm{i}k_\alpha^i\partial_i K - \tfrac{1}{2}\mathrm{i}k_\alpha^{\bar{i}}\partial_{\bar{i}}K + g\xi_\alpha = \left(\tfrac{1}{2}\mathrm{i}k_\alpha^A\partial_A\mathcal{K} - \tfrac{1}{2}\mathrm{i}k_\alpha^{\bar{A}}\partial_{\bar{A}}K\right)_{\mathcal{K}=-3M_P{}^2}. \end{aligned} \tag{5.5}$$

In the first expression for $\mathcal{P}$ occur the Killing vectors in the directions of the physical scalars k_α^i and the Fayet–Iliopoulos term ξ_α. The latter is in the conformal formulation related to the component of the Killing vector in the direction of Y, i.e. $k_\alpha^Y = \mathrm{i}g\xi_\alpha Y/\left(3M_P^2\right)$.

6. Final remarks

We have given an overview of a landscape of possible supergravity theories. Consistency with supergravity gives also many restrictions on an effective field theory of the fields near a string theory vacuum. We have illustrated here some general features of supergravity theories, and given the key ingredients of the superconformal formulation that leads to insights in their structure.

This has been used to construct the effective $N=1$ supergravity theory of cosmic strings in [11]. The fact that this construction is embedded in the basic $N=1$ theory may look as a shortcoming. However, often such models can be embedded in larger supergravity theories.

For example, the mentioned cosmic string configuration could be embedded in an $N = 2$ supergravity [21] in a way such that effectively only some fields of the larger theory are non-vanishing, which brings us back to the $N = 1$ set-up. Such embeddings are, however, non-trivial. There is a main consistency requirement that the reduction of the field equations of the $N = 2$ theory to the subsector gives the same result as calculating the field equations of the reduced Lagrangian. $N = 2$ consistent truncations have been considered in detail in, e.g., [22, 23]. In geometrical terms it says that the submanifold must be 'geodesic'. This means that any geodesic in the submanifold should be a geodesic of the ambient manifold.

Such principles also hold when one considers a small set of multiplets in a supergravity and wants to consider it within a larger model with a larger set of multiplets. For example, recently we investigated how several $N = 2$ supergravities with special geometry can be embedded in each other, such that a solution of the smallest one can be taken over as a solution of the larger one. That leads to a restricted set of basic homogeneous special geometries [24].

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